# Exploratory biomass model assessment of the Icelandic slope redfish (Sebastes mentella) 

Arni Magnusson

22 January 2012


#### Abstract

Two biomass models are fitted to the Icelandic slope redfish stock: the Schaefer model and the replacement yield model. Both models lead to the same conclusions, with point estimates of interest including $B_{1978}=1406$ and $B_{2010}=544$ thousand t. The productivity of the stock is estimated to be very low, with annual surplus production more likely to be below 10 thousand $t$ than above it. Uncertainty analysis based on profile likelihood shows great uncertainty about all estimated quantities.


## Contents

1 Introduction ..... 2
2 Data ..... 3
3 Models ..... 5
3.1 General approach ..... 5
3.2 Schaefer ..... 5
3.3 Replacement yield ..... 5
4 Results ..... 6
4.1 Model fit ..... 6
4.2 Point estimates ..... 6
4.3 Uncertainty ..... 7
5 Discussion ..... 8
6 References ..... 9

## 1 Introduction

Relatively little is known about the status of the Icelandic slope redfish stock, in terms of current or historical biomass levels. The annual report (MRI 2011) presents no formal assessment, only landings from 1978 and biomass indices from the autumn survey since 2000. The biomass indices fluctuate between years but indicate a decline of around $30 \%$ from 2000 to 2010, although the annual landings since 2000 amount to only half of the annual landings during the 1990s (Table 1, Figs 1 and 2).

The existence of catch and biomass index time series prompted the author to try to fit a biomass model to the data. The main objective is to quantify the uncertainty about current biomass and depletion level, i.e. explore the range of possible values, given the limited data and applying the restrictive assumptions of a biomass model. Two biomass models are considered: the logistic model (Schaefer 1954) and the replacement yield model (Butterworth and Geromont 1996).

## 2 Data

Table 1. Landings, autumn survey biomass index, and nominal coefficient of variation.

|  | Catch (1000 t) | Index | CV |
| :---: | :---: | :---: | :---: |
| 1978 | 3.902 |  |  |
| 1979 | 7.694 |  |  |
| 1980 | 10.197 |  |  |
| 1981 | 19.689 |  |  |
| 1982 | 18.492 |  |  |
| 1983 | 37.115 |  |  |
| 1984 | 24.493 |  |  |
| 1985 | 24.768 |  |  |
| 1986 | 18.898 |  |  |
| 1987 | 19.293 |  |  |
| 1988 | 14.290 |  |  |
| 1989 | 40.269 |  |  |
| 1990 | 28.429 |  |  |
| 1991 | 47.651 |  |  |
| 1992 | 43.414 |  |  |
| 1993 | 51.221 |  |  |
| 1994 | 56.720 |  |  |
| 1995 | 48.708 |  |  |
| 1996 | 34.741 |  |  |
| 1997 | 37.876 |  |  |
| 1998 | 33.125 |  |  |
| 1999 | 28.590 |  |  |
| 2000 | 31.393 | 138.9 | 0.145 |
| 2001 | 17.230 | 164.0 | 0.172 |
| 2002 | 19.045 | 96.9 | 0.137 |
| 2003 | 28.478 | 64.6 | 0.127 |
| 2004 | 17.564 | 98.4 | 0.164 |
| 2005 | 20.563 | 114.9 | 0.249 |
| 2006 | 17.208 | 124.5 | 0.172 |
| 2007 | 17.373 | 85.5 | 0.183 |
| 2008 | 24.123 | 82.7 | 0.139 |
| 2009 | 19.430 | 99.7 | 0.183 |
| 2010 | 17.668 | 81.8 | 0.149 |



Figure 1. Landings.


Figure 2. Autumn survey biomass index, with nominal standard error (lognormal).

## 3 Models

### 3.1 General approach

The biomass in the first year is an estimated parameter $B_{\text {init }}$. In subsequent years,

$$
\begin{equation*}
B_{t+1}=B_{t}+g\left(B_{t}\right)-C_{t} \tag{1}
\end{equation*}
$$

where $g\left(B_{t}\right)$ is a surplus production function (see next subsection) and $C_{t}$ is catch.
The biomass index is predicted as

$$
\begin{equation*}
\hat{I}_{t}=q B_{t} \tag{2}
\end{equation*}
$$

where $q$ is a catchability coefficient.
Assuming lognormal uncertainty, the objective function is

$$
\begin{equation*}
-\log L=0.5 n \log (2 \pi)+\sum \log \sigma_{t}+\sum \frac{\left(\log I_{t}-\log \hat{I}_{t}\right)^{2}}{2 \sigma_{t}^{2}} \tag{3}
\end{equation*}
$$

The data come with nominal coefficients of variation that describe the uncertainty about the observed biomass indices. These are not used as absolute $\sigma_{t}$ in the objective function, but rather as relative coefficients $\xi_{t}$ that are multiplied with an estimated scaler $\tau$ to predict $\sigma_{t}$ :

$$
\begin{equation*}
\sigma_{t}=\xi_{t} \tau \tag{4}
\end{equation*}
$$

### 3.2 Schaefer

The Schaefer (1954) model models annual surplus production with two parameters,

$$
\begin{equation*}
g(B)=r B_{t}\left(1-\frac{B_{t}}{K}\right) \tag{5}
\end{equation*}
$$

where $r$ is maximum growth rate and $K$ is carrying capacity.
The population is assumed to be at carrying capacity in the first year,

$$
\begin{equation*}
B_{\mathrm{init}}=K \tag{6}
\end{equation*}
$$

resulting in four estimated parameters: $r, K, q, \tau$.

### 3.3 Replacement yield

The replacement yield model (Butterworth and Geromont 1996) models annual surplus production with one simple parameter,

$$
\begin{equation*}
g(B)=\gamma \tag{7}
\end{equation*}
$$

where the annual surplus production $\gamma$ is the same in all years, with no density dependence. During estimation, $\gamma$ is constrained to be positive.

Four estimated parameters: $B_{\text {init }}, \gamma, q, \tau$.

## 4 Results

### 4.1 Model fit

The two biomass models produce perfectly identical biomass predictions, and therefore fit the data equally well (Fig. 3) using the same number of parameters.


Figure 3. Model fit to autumn survey biomass index. The Schaefer and replacement yield models have the same fit.

### 4.2 Point estimates

Both models fit the data best when annual surplus production is zero, in the form of $r=0$ and $\tau=0$ (Table 2).

Table 2. Parameter estimates from the two biomass models.

| Model | Parameter estimates |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Schaefer | $K=1406$ | $r=0.000$ | $q=0.151$ | $\tau=1.54$ |  |  |  |
| Replacement yield | $B_{\text {init }}=1406$ | $\gamma=0.000$ | $q=0.151$ | $\tau=1.54$ |  |  |  |

The estimated biomass in the last year, $B_{2010}$, is estimated as 544 thousand t , corresponding to a depletion level of $B_{2010} / B_{\text {init }}=0.39$.

### 4.3 Uncertainty

To quantify the uncertainty about the current biomass and depletion level, profile likelihood was applied to the replacement yield model. The Schaefer model was also analyzed in the same way, giving very similar results, but less numerically stable.

The $B_{\text {init }}$ parameter was fixed at various arbitrary values, while estimating the $\gamma, q$ and $\tau$ to maximize the likelihood. The resulting profiles indicate a high level of uncertainty for estimated quantities like current biomass (Fig. 4), depletion level (Fig. 5), and annual surplus production (Fig. 6).


Figure 4. Profile likelihood for current biomass.


Figure 5. Profile likelihood for current depletion level.


Figure 6. Profile likelihood for annual surplus production.

A $95 \%$ confidence interval covers the profile likelihood interval where $L>0.15$, or $L>e^{-0.5 \chi_{d f=1}^{2}}$ to be exact (Hilborn and Mangel 1997). There is no upper limit in the $95 \%$ confidence intervals for $B_{\text {init }}$ and $B_{2010}$ (Table 3), because a flat horizontal line fits the observed biomass indices with $L=0.37$. The most likely value of $\gamma$ is zero, but higher values of $\gamma$ become likely if the biomass is very small, with the most extreme case being $B_{2010}=40, \gamma=21$.

Table 3. $95 \%$ confidence intervals and maximum likelihood estimate of selected estimated quantities, based on profile likelihood.

| Quantity | Lower | MLE | Upper |
| :--- | ---: | ---: | ---: |
| $B_{\text {init }}$ | 245 | 1406 | $\infty$ |
| $B_{2010}$ | 40 | 544 | $\infty$ |
| $B_{2010} / B_{\text {init }}$ | 0.16 | 0.39 | 1.00 |
| $\gamma$ | 0 | 0 | 21 |

## 5 Discussion

Both the Schaefer and replacement yield biomass models tell a similar story. The most likely parameter values describe a stock with very low productivity, which has been gradually mined down for three decades, but the biomass is still relatively large, around $40 \%$ of carrying capacity.

The uncertainty analysis highlights two possible extremes. One possibility is that the stock may be considerably larger than the MLE, but still with very low productivity. Another possibility is that the stock is considerably smaller than the MLE, with productivity that comes close to sustaining the annual removals, but in that case the depletion level could be as low as $19 \%$ of carrying capacity.

Altogether, this study puts in numbers what was already known: there is great uncertainty about the stock status of the Icelandic slope redfish, given the landings and autumn survey time series. Other data components, such as length and age distribution may provide additional information about productivity (recruitment) and stock status.

## 6 References

Butterworth, D. and H. Geromont. 1996. Replacement yield calculations for the South Coast rock lobster. MCM WG/07/96/SCL10.
Hilborn, R. and M. Mangel. 1997. The ecological detective: Confronting models with data. Princeton: Princeton University Press.
MRI (Marine Research Institute). 2011. State of marine stocks in Icelandic waters 2010/2011 and prospects for the quota year 2011/2012]. Marine Research 159.
Schaefer, M.B. 1954. Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. IATTC Bull. 1:27-56.

