Mathematical properties of the Icelandic saithe HCR

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1   When SSB $\geq$ B$_{\text{trigger}}$

Official HCR equation (Ministry of Industries and Innovation 2013, Equation 1):

$$TAC_{y/y+1} \quad = \quad \big(1-\beta\big)\alpha\tilde{B}_{4+,y} \quad + \quad \beta\,TAC_{y-1/y} \tag{1.a}$$

Simplify notation,

$$TAC_t \quad = \quad \big(1-\beta\big)\alpha B_{t,4+} \quad + \quad \beta\,TAC_{t-1} \tag{1.b}$$

replace $\alpha = 0.20$ and $\beta = 0.5$:

$$TAC_t \quad = \quad 0.5 \times 0.20\,B_{t,4+} \quad + \quad 0.5\,TAC_{t-1}$$

$$= \quad \frac{0.20\,B_{t,4+} \; + \; TAC_{t-1}}{2} \tag{1.c}$$

A plain average of two numbers.

2   When SSB $<$ B$_{\text{trigger}}$

Official HCR equation (Ministry of Industries and Innovation 2013, Equation 2):

$$TAC_{y/y+1} \quad = \quad \big(1 - \beta\tfrac{S\tilde{S}B_y}{B_{\text{trigger}}}\big)\alpha\tfrac{S\tilde{S}B}{B_{\text{trigger}}}\,\tilde{B}_{4+,y} \quad + \quad \beta\tfrac{S\tilde{S}B}{B_{\text{trigger}}}\,TAC_{y-1/y} \tag{2.a}$$

Simplify notation,

$$TAC_t \quad = \quad \big(1 - \beta\tfrac{SSB_t}{B_{\text{trigger}}}\big)\alpha\tfrac{SSB_t}{B_{\text{trigger}}}\,B_{t,4+} \quad + \quad \beta\tfrac{SSB_t}{B_{\text{trigger}}}\,TAC_{t-1} \tag{2.b}$$

replace $\alpha = 0.20$ and $\beta = 0.5$,

$$TAC_t \quad = \quad \big(1 - 0.5\tfrac{SSB_t}{B_{\text{trigger}}}\big)0.20\tfrac{SSB_t}{B_{\text{trigger}}}\,B_{t,4+} \quad + \quad 0.5\tfrac{SSB_t}{B_{\text{trigger}}}\,TAC_{t-1} \tag{2.c}$$

and rearrange:

$$TAC_t \quad = \quad \tfrac{SSB_t}{B_{\text{trigger}}}\left[\big(1 - 0.5\tfrac{SSB_t}{B_{\text{trigger}}}\big)0.20\,B_{t,4+} \quad + \quad 0.5\,TAC_{t-1}\right] \tag{2.d}$$

3   Desired properties

3.1   Continuity

Since SSB can be expected to go below B_{trigger} every once in a while (5% of the time), it is desirable to have a rule that is not overly sensitive to whether SSB is just below or above B_{trigger}. In other words, the rule should not lead to wildly different TAC when $\frac{SSB_t}{B_{\text{trigger}}}$ is 0.999 or 1.001.

When $\frac{SSB_t}{B_{\text{trigger}}}$ tends towards 1, Equation (2.d) converges to the plain average:

$$
\begin{aligned}
TAC_t \ &= \ \tfrac{SSB_t}{B_{\text{trigger}}} \left[\left(1 - 0.5 \tfrac{SSB_t}{B_{\text{trigger}}}\right) 0.20\, B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \right] \\[2mm]
&= \ \left(1 - 0.5\right) 0.20\, B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \\[2mm]
&= \ 0.5 \times 0.20\, B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \\[2mm]
&= \ \frac{0.20\, B_{t,4+} \ + \ TAC_{t-1}}{2}
\end{aligned}
\tag{3.a}
$$

3.2   Precaution

It is also desirable to have rule that quickly brings the harvest rate down when SSB is very low, even if TAC_{t-1} was high.

When $\frac{SSB_t}{B_{\text{trigger}}}$ tends to 0, Equation (2.d) converges to zero:

$$
\begin{aligned}
TAC_t \ &= \ \tfrac{SSB_t}{B_{\text{trigger}}} \left[\left(1 - 0.5 \tfrac{SSB_t}{B_{\text{trigger}}}\right) 0.20\, B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \right] \\[2mm]
&= \ 0 \times \left[0.20\, B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \right] \\[2mm]
&= \ 0
\end{aligned}
\tag{3.b}
$$

4   Discussion

The official HCR corresponds to 'Rule 4' in the HCR evaluation (Hjorleifsson and Bjornsson 2013). Another option, not included in the HCR evaluation, would have been to simply scale the plain average,

$$
\begin{aligned}
TAC_t \ &= \ \tfrac{SSB_t}{B_{\text{trigger}}} \left[(1-\beta)\alpha B_{t,4+} \ \ + \ \ \beta\, TAC_{t-1} \right] \\[2mm]
&= \ \tfrac{SSB_t}{B_{\text{trigger}}} \left(0.5 \times 0.20 B_{t,4+} \ \ + \ \ 0.5\, TAC_{t-1} \right) \\[2mm]
&= \ \tfrac{SSB_t}{B_{\text{trigger}}} \left(\frac{0.20 B_{t,4+} \ + \ TAC_{t-1}}{2} \right)
\end{aligned}
\tag{4.a}
$$

which fulfills both of the above properties, as well as the desired property of simplicity.

The difference between the official HCR (Equation 2.d) and the simple alternative (Equation 4.a) is that the $\left(1-0.5\frac{SSB_t}{B_{\text{trigger}}}\right)$ term is replaced with 0.5 in the simple version.

Since $\frac{SSB_t}{B_{\text{trigger}}} < 1$, the $\left(1-0.5\frac{SSB_t}{B_{\text{trigger}}}\right)$ term will be greater than 0.5, so the official HCR is less precautious than the simple version: the HCR does not reduce the harvest rate as quickly when SSB goes below B_{trigger}.

Since SSB is not expected to go far below B_{trigger} for many years, the official HCR and the simple version are effectively the same for the purposes of managing the Icelandic saithe fishery.

References

Hjorleifsson, E. and H. Bjornsson. 2013. Evaluation of the Icelandic saithe management plan. ICES CM 2013/ACOM:60.

Minstry of Industries and Innovation. 2013. Adoption of management plan for Iceland saithe. Letter to ICES, dated 22 April 2013. Ref: ANR12090104/11.2.3.