Estimation of Fish Growth Curves

Arni Magnusson

Research Seminar
UiT The Arctic University of Norway
25 February 2026

My background

Ph.D. in fisheries science, specializing in statistical computing

2000 – 2016	Scientist in Iceland, Seattle, and New Zealand
2016 – 2026	International organizations: ICES (North Atlantic), FAO (Mediterranean), and SPC (Pacific Ocean)

My research has focused on statistical methods
and development of tools in data science

Areas of interest and expertise:

- nonlinear models (using AD Model Builder/TMB/RTMB, glmmTMB)
- evaluation of uncertainty (delta method, bootstrap, profile likelihood, MCMC)
- model ensembles, high-throughput computing clusters (Condor)
- tools that support open, repeatable, and reviewable science (TAF, makeit, xtable)

University course that I taught last month

1. **Introduction to R**

   data, plots, tests, linear models, projects, help, functions, packages

2. **Extending the Linear Model**

   generalized linear models, additive models, mixed effects

3. **Nonlinear Models**

   uncertainty, maximum likelihood, hessian, simulations

4. **RTMB**

   automatic differentiation, writing models, running models

5. **Software Development**

   interacting with other programs, writing packages, github

Overview of today's talk

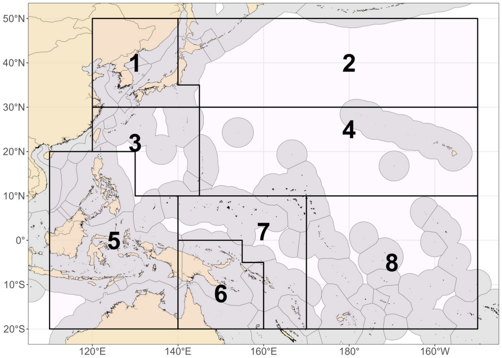
Fish Populations *population dynamics, length and age*

Growth Curves *von Bertalanffy, Gompertz, Richards, size variability*

The fishgrowth Package in R *overview, features, examples*

Nonlinear Models *likelihood, automatic differentiation, random effects, automatic sparsity detection, delta method, RTMB*

Marine Science and the Pacific Ocean



Population dynamics

Year	Age 1	Age 2	Age 3	Age 4
2010	2912	307	62	49
2011	1789	359	87	37
2012	2972	217	101	40
2013	2911	343	62	46
2014	2897	339	96	35
2015	2823	354	94	43
2016	2246	360	100	45
2017	2330	255	100	47
2018	2862	242	71	48
2019	2692	326	66	39
2020	2580	346	89	34
2021	2347	253	96	39
2022	2518	305	70	44
2023	2636	291	90	38
2024	2611	300	80	42

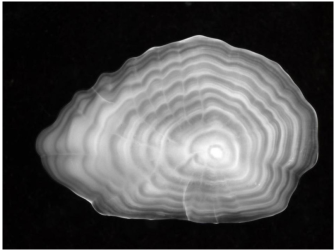
We keep track of the population size in numbers by age and year

Cohorts become smaller with age

For the rest of the talk, we focus on growth curves, a subcomponent of a full population dynamics model

Population models are often age-structured, but age can be challenging to measure directly

Otoliths and age determination



Otolith from haddock

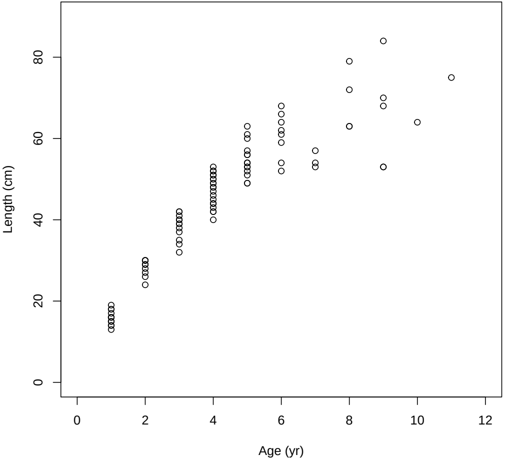
In temperate and higher latitudes, fish grow fast during the summer and slow during the winter

This results in otolith year rings that reflect their age, like tree trunks

Age determination is much harder in the tropics, where fish grow steadily throughout the year, since there is little difference between seasons

Scientists have recently attempted to count daily rings in young tropical fish, as they grow fast during the day and slow during the night

Otolith data and growth



We can estimate a growth curve from otolith data

Growth curves

von Bertalanffy

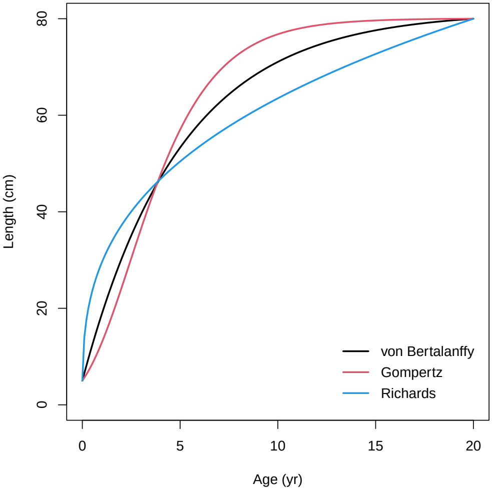
$$L_t = L_\infty \left(1 - e^{-k(t-t_0)}\right)$$

Gompertz

$$L_t = L_\infty \exp\left(-e^{-k(t-\tau)}\right)$$

Richards

$$L_t = L_\infty \left(1 - \frac{1}{b} e^{-k(t-\tau)}\right)^b$$



Reparametrization

von Bertalanffy $(b = 1)$

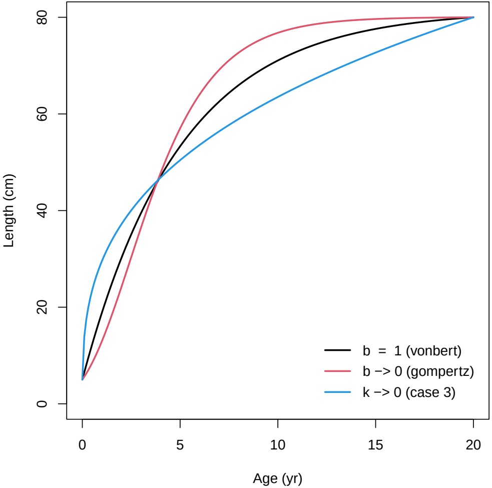
$$L_t = L_1 + (L_2 - L_1)\,\frac{1 - e^{-k(t - t_1)}}{1 - e^{-k(t_2 - t_1)}}$$

Gompertz $(b \to 0)$

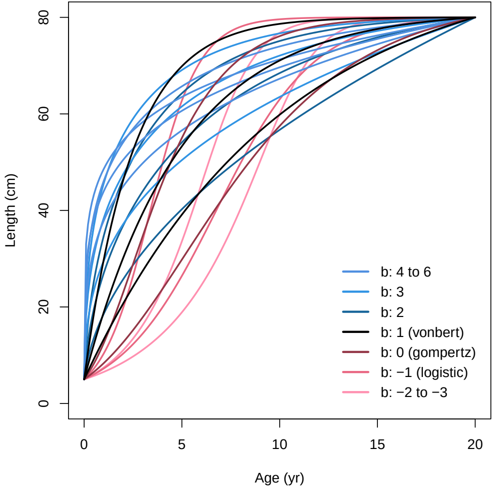
$$L_t = L_1 \exp\left[\log(L_2/L_1)\,\frac{1 - e^{-k(t - t_1)}}{1 - e^{-k(t_2 - t_1)}}\right]$$

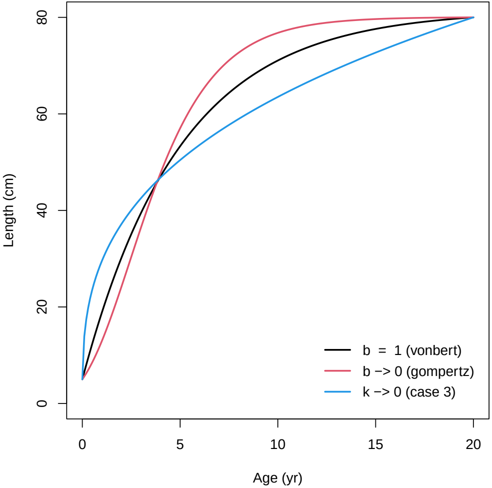
Richards

$$L_t = \left[L_1^b + (L_2^b - L_1^b)\,\frac{1 - e^{-k(t - t_1)}}{1 - e^{-k(t_2 - t_1)}}\right]^{1/b}$$



Richards curves





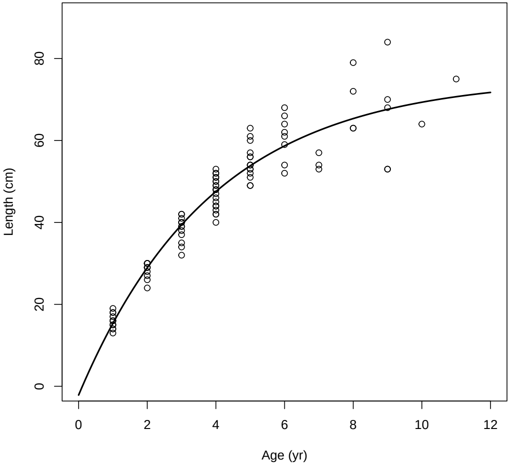
Fitting a model

von Bertalanffy curve

$$\hat{L}_i = L_\infty \left(1 - e^{-k(t_i - t_0)}\right)$$

Negative log-likelihood

$$f = \sum_{i=1}^{N_{\text{oto}}} 0.5 \log(2\pi) + \log \sigma_i + \frac{(L_i - \hat{L}_i)^2}{2\sigma_i^2}$$



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otoliths <- read.csv("haddock.csv")

par <- list(Linf=100, k=1, t0=0, sigma=1)

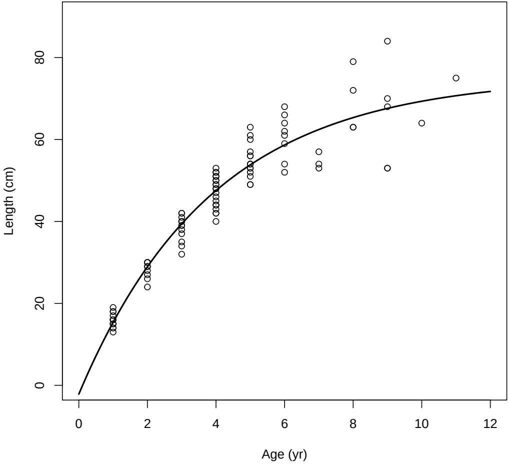
f <- function(par)
{
  Linf <- par[["Linf"]]
  k <- par[["k"]]
  t0 <- par[["t0"]]
  sigma <- par[["sigma"]]

  t <- otoliths$age
  L <- otoliths$len

  Lhat <- Linf * (1 - exp(-k*(t-t0)))
  nll <- -sum(dnorm(L, Lhat, sigma, TRUE))
  nll
}


fit <- nlminb(par, f)
```

Fitting a model

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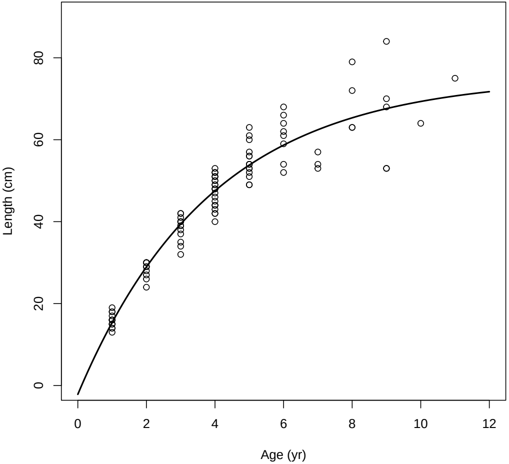
Size variability



σ increases with length

Overview

Fish Populations *population dynamics, length and age*

Growth Curves *von Bertalanffy, Gompertz, Richards, size variability*

The fishgrowth Package in R *overview, features, examples*

Nonlinear Models *likelihood, automatic differentiation, random effects, automatic sparsity detection, delta method, RTMB*

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- You Are Here

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Package ‘fishgrowth’

November 7, 2025

Version 1.0.4

Title Fit Growth Curves to Fish Data

Depends R (>= 2.10), RTMB

Description Fit growth models to otoliths and/or tagging data, using the 'RTMB' package and maximum likelihood. The otoliths (or similar measurements of age) provide direct observed coordinates of age and length. The tagging data provide information about the observed length at release and length at recapture at a later time, where the age at release is unknown and estimated as a vector of parameters. The growth models provided by this package can be fitted to otoliths only, tagging data only, or a combination of the two. Growth variability can be modelled as constant or increasing with length.

License GPL-3

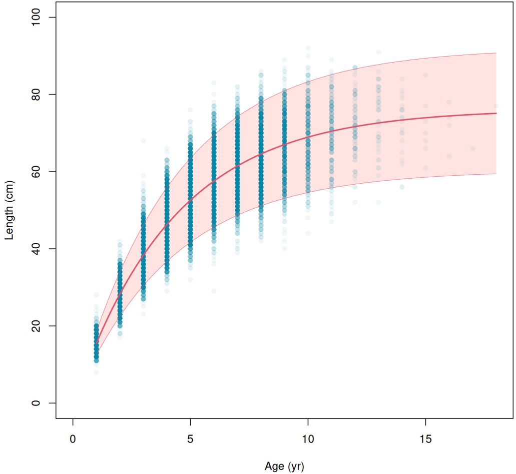
URL https://github.com/arni-magnusson/fishgrowth

Author Arni Magnusson [aut, cre], Mark Maunder [aut]

Maintainer Arni Magnusson <thisisarni@gmail.com>

Repository CRAN

Haddock example



The prediction band describes the variability in length-at-age

Von Bertalanffy curve,
σ increases with length

5 parameters:

$L_1, L_2, k, \sigma_{\min}, \sigma_{\max}$

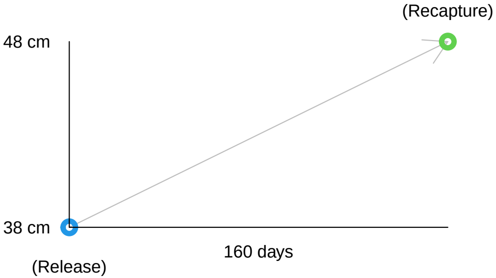
Icelandic haddock from the Icelandic spring bottom trawl survey 2011–2020

$n = 40,364$

Tagging data and growth increment

A fish is tagged, measured, and released

When a tagged fish is recaptured, it is measured again

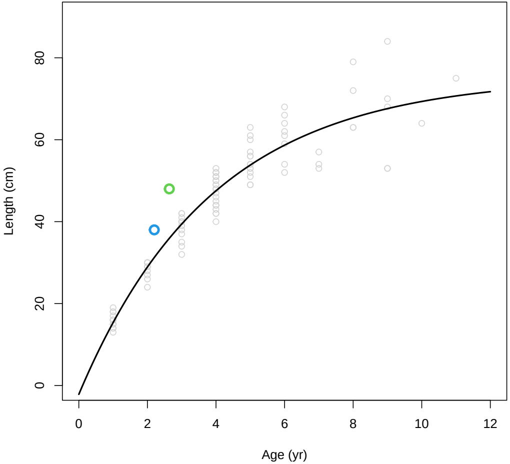


We know the

- length at release
- length at recapture
- time difference (time at liberty)

but not the actual age

Estimating growth from tagging data

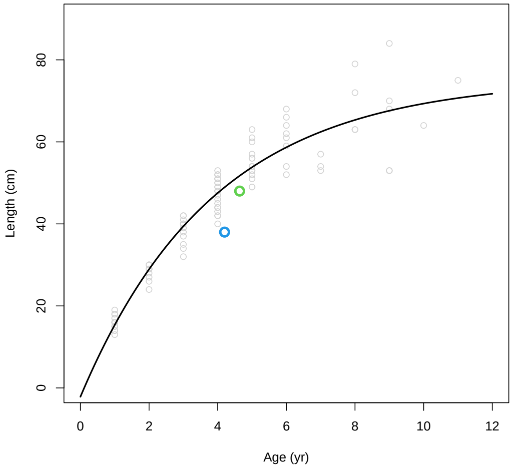


Each tag has two points:
release and **recapture**

For each point pair:

- *y*-coordinates are known
  - *e.g., 38 cm and 48 cm*
- *x*-distance between them is known
  - *e.g., 160 days*
- location on the *x*-axis can be estimated

Estimating growth from tagging data



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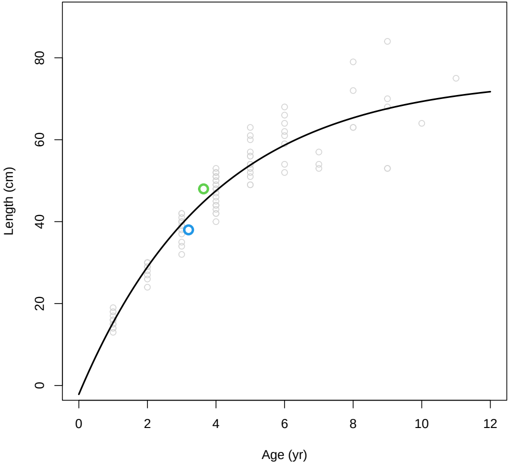

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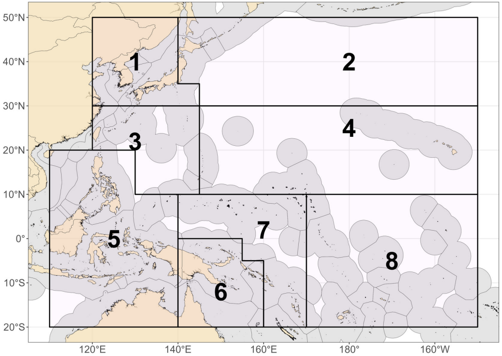
Estimating growth from tagging data

The age at release is estimated as a free parameter for every tag

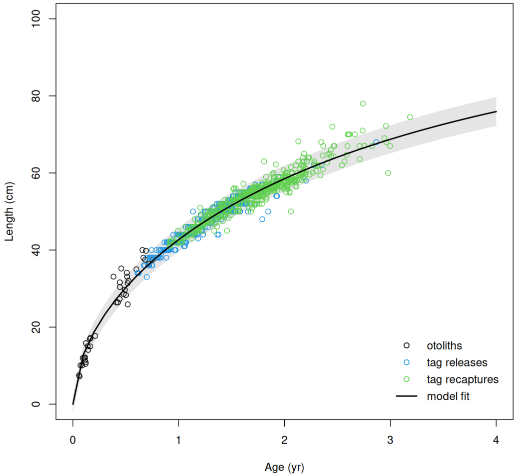
The objective function becomes

$$
\begin{aligned}
f \;=\; & \sum_{i=1}^{N_{\mathrm{oto}}} 0.5\log(2\pi) \;+\; \log\sigma_i \;+\; \frac{(L_i - \hat{L}_i)^2}{2\sigma_i^2} && \text{otoliths} \\
+\; & \sum_{j=1}^{N_{\mathrm{tag}}} 0.5\log(2\pi) \;+\; \log\sigma_j \;+\; \frac{(L_j - \hat{L}_j)^2}{2\sigma_j^2} && \text{tag releases} \\
+\; & \sum_{k=1}^{N_{\mathrm{tag}}} 0.5\log(2\pi) \;+\; \log\sigma_k \;+\; \frac{(L_k - \hat{L}_k)^2}{2\sigma_k^2} && \text{tag recaptures}
\end{aligned}
$$

Skipjack tuna in the western and central Pacific



Tuna example



Otoliths are only available for the youngest ages

Richards model

493 parameters

L_1, L_2, k, b, σ_{min}, σ_{max},

age at release for every tag

Skipjack tuna data from the western and central Pacific Ocean 2009–2021

$n_{\text{oto}} = 39$ *otoliths*

$n_{\text{tag}} = 487$ *tags*

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$\rightarrow$

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Nonlinear models

Optimization Methods & Software
Vol. 27, No. 2, April 2012, 233–249

AD Model Builder: using automatic differentiation for statistical inference of highly parameterized complex nonlinear models

David A. Fournier[a], Hans J. Skaug[b]*, Johnoel Ancheta[c], James Ianelli[d], Arni Magnusson[e], Mark N. Maunder[f], Anders Nielsen[g] and John Sibert[c]

FISH and FISHERIES, 2013, **14**, 325–342

Measuring uncertainty in fisheries stock assessment: the delta method, bootstrap, and MCMC

Arni Magnusson[1,2], André E Punt[1] & Ray Hilborn[1]

Methods in Ecology and Evolution 2013, **4**, 501–512

Strategies for fitting nonlinear ecological models in R, AD Model Builder, and BUGS

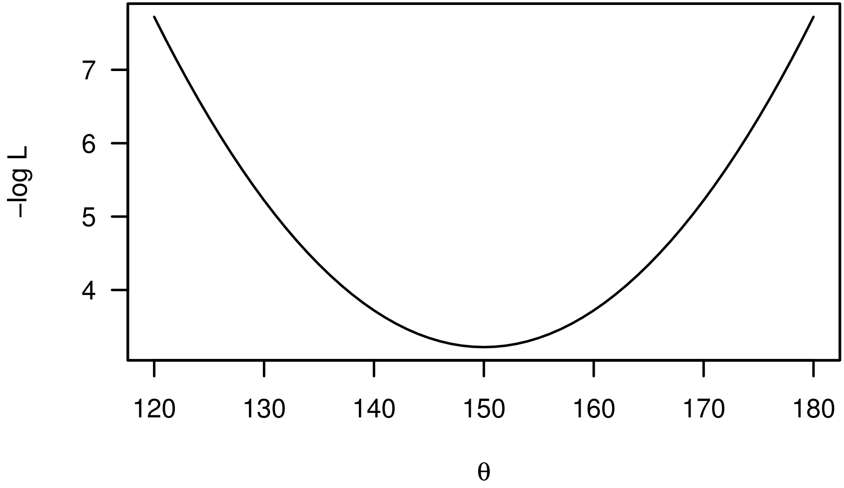
Benjamin M. Bolker[1]*, Beth Gardner[2] †, Mark Maunder[3], Casper W. Berg[4], Mollie Brooks[5], Liza Comita[6] ‡, Elizabeth Crone[7], Sarah Cubaynes[8] §, Trevor Davies[9], Perry de Valpine[10], Jessica Ford[11], Olivier Gimenez[8], Marc Kéry[12], Eun Jung Kim[13], Cleridy Lennert-Cody[3], Arni Magnusson[14], Steve Martell[15], John Nash[16], Anders Nielsen[4], Jim Regetz[6], Hans Skaug[17] and Elise Zipkin[2]

The R Journal Vol. 9/2, December 2017

glmmTMB Balances Speed and Flexibility Among Packages for Zero-inflated Generalized Linear Mixed Modeling

by Mollie E. Brooks, Kasper Kristensen, Koen J. van Benthem, Arni Magnusson, Casper W. Berg, Anders Nielsen, Hans J. Skaug, Martin Mächler, Benjamin M. Bolker

Likelihood



Automatic differentiation

Journal of Statistical Software

April 2016, Volume 70, Issue 5. doi: 10.18637/jss.v070.i05

TMB: Automatic Differentiation and Laplace Approximation

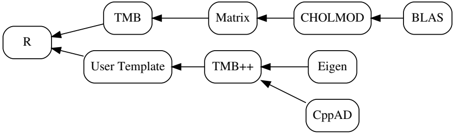
Kasper Kristensen DTU Compute

Anders Nielsen DTU Aqua

Casper W. Berg DTU Aqua

Hans Skaug University of Bergen

Bradley M. Bell University of Washington



random effects, automatic sparsity detection, delta method

Overloaded C++ operators are used to store derivatives in all calculations

Numbers are special objects with derivative information stored as a class property

Standard calculus rules are used to calculate all derivatives, using the chain rule, etc.

All of this is handled internally, without the user having to think about it

RTMB is the current recommended user interface for developing and running models

R

```r
otoliths <- read.csv("haddock.csv")

par <- list(Linf=100, k=1, t0=0, sigma=1)

f <- function(par)
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fit <- nlminb(par, f)
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RTMB

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library(RTMB)

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  nll
}

model <- MakeADFun(f, par)
fit <- nlminb(model$par, model$fn, model$gr)
rep <- summary(sdreport(model))
```

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First steps as faculty

Establish collaboration with UiT colleagues across multiple departments

Write and submit journal articles describing software that I have created:

fishgrowth	growth curve estimation
r2d2	bivariate confidence regions
fishstat	global fisheries data from FAO for open data science
TAF	framework for open science, repeatable and reviewable

Collaborate with my network of statisticians (expecially in Denmark and Norway) as a coauthor, contributing my expertise in statistical software development

Launch a project to apply the newest methodology to analyze the status of fisheries in developing countries, leveraging my network at FAO and selected scientists, including alumni that I have mentored and collaborated with, who now work in their home countries in Africa, Asia, Pacific Islands, and the Caribbean